Review Worksheet 4

Areas under the SNC

1. The current mean height ($\mu$) of men in the U.S. that are $\geq$20 years of age is 69.5 inches with a standard deviation ($\sigma$) of 2.8 inches.

If you were to randomly choose one man 20 years or older in the U.S.:

a) What is the probability he would be $\leq$ 65 inches in height?

$$z = \frac{x - \mu}{\sigma} = \frac{65 - 69.5}{2.8} = -1.61$$

$z_{-1.61} =$ Area of 0.0537

Because we want “less than” a value, do not subtract from one. The chance he would be $\leq$ 65 inches in height is 5.37%.

b) What is the probability he would be between 68 and 71 inches in height?

$$z_{0.54} = \frac{68 - 69.5}{2.8} = -0.54$$

$z_{0.54} =$ Area of 0.2946

$$z_{0.54} = \frac{71 - 69.5}{2.8} = 0.54$$

$z_{0.54} =$ Area of 0.7054

0.7054 – 0.2946 = 0.4108

41.08% chance of being between 68 and 71 inches

C) What is the probability she would be $\geq$76 inches in height?

$$z = \frac{76 - 69.5}{2.8} = 2.32$$

$z_{2.32} =$ Area of 0.9898

Because we want “greater than” a value, subtract area from 1.0.

1 – 0.9898 = 0.0102

1.02% chance of being $\geq$76 inches in height.

d) Based on this data, what is the exact height that corresponds to the top 10% of males by height?

First, convert % into an area. 10% is an area (or probability) of 0.10. Because we want the top or “greater than”, subtract area from 1.0.

1 – 0.10 = 0.90

Now, convert this area into a $z$-score. Area of 0.90 = $z_{1.28}$ (closest area in table to 0.90 is 0.8997). So $z = 1.28$.

$$x = (z \times \sigma) + \mu$$

$$x = (1.28 \times 2.8) + 69.5 = 3.58 + 69.5 = 73.1$$

73.1 inches is the cutoff for the top 10% of men by height.
**Confidence Intervals**

2. I randomly chose 30 male heights (I estimated height from the first 30 men on my cell phone contact list). From this sample, I calculated a sample mean ($\overline{x}$) of 70.5 and a sample SD ($s$) of 2.3.

a) Calculate a 90% CI using population SD, $\sigma$.

\[
\overline{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}
\]

\[
70.5 \pm 1.64 \left( \frac{2.8}{\sqrt{30}} \right)
\]

\[
70.5 \pm 0.84
\]

90% Confidence Interval (CI) is **69.7 to 71.3 inches**

b) Calculate a 95% CI using sample SD, $s$.

\[
\overline{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}
\]

\[
70.5 \pm 2.045 \left( \frac{2.3}{\sqrt{30}} \right)
\]

\[
70.5 \pm 0.86
\]

95% Confidence Interval (CI) is **69.6 to 71.4 inches**
Hypothesis Tests

3. The current mean height (\( \mu \)) of women in the U.S. that are \( \geq 20 \) years of age is 64.0 inches with a standard deviation (\( \sigma \)) of 2.8 inches.

I randomly chose 30 female heights (I estimated height from the first 30 women on my cell phone contact list). From this sample, I calculated a sample mean (\( \bar{x} \)) of 65.5 and a sample SD (\( s \)) of 2.5.

Using a significance level of 10\% (\( \alpha = 0.10 \)) and assuming the SD is the same for both populations (use \( \sigma = 2.8 \)), are we very confident that the mean height is the same for women as for men?

Our hypothesis is that our women’s sample mean height (65.5) is statistically the same as that of men (69.5). Obviously, this is probably not the case, but we will test our hypothesis. Our critical value of \( z \) will be 1.64 (90\% confidence).

\[
z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{65.5 - 69.5}{2.8/\sqrt{30}} = \frac{-4.0}{0.51} = -7.84
\]

This value is very much below our critical \( z \)-value of -1.64, so we reject the hypothesis that woman are as tall as men.
4. I randomly chose 30 female heights (I estimated height from the first 30 women on my cell phone contact list). From this sample, I calculated a sample mean ($\bar{x}$) of 65.5 and a sample SD ($s$) of 2.5.

Using a significance level of 5% ($\alpha = 0.05$) and sample SD ($s$), are we very confident that the mean height is the same for women who I am acquainted with compared to the total population of women in the U.S.?

Our critical value of $t$ will be $2.045$ ($df = (n - 1) = 29$ and column $t_{0.025}$).

$$ t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{65.5 - 64.0}{2.5/\sqrt{30}} = \frac{1.5}{0.45} = 3.3 $$

This value is above our critical $t$-value of 2.045, so we reject the hypothesis that height of my female friends is the same as the general population of women in the U.S.